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CONSTRAINED BÉZIER CURVES

In this paper we present two new classes of nth degree Bézier curves while its control points $\mathbf{b}_{n-1}$ or $\mathbf{b}_1$ are always belong to a curve. Degenerate Bézier curves may be considered as a particular case of presented class.

A Bézier curve is a parametric curve frequently used in computer graphics and related fields to model smooth curves that can be scaled indefinitely.

As usual it is written as follows

$$\mathbf{b}^n(t) = \sum_{i=0}^{n} \mathbf{b}_i B_i^n(t), \tag{1}$$

where $B_i^n(t)$ are Bernstein polynomials, defined explicitly by

$$B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}.$$

Let us consider that a curve (1) has a control point $\mathbf{b}_{n-1} \in \mathbf{b}^n(t)$ for all $t \in [0,1]$, then it is possible to write

$$\sum_{i=0}^{n} \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} = \mathbf{b}_{n-1},$$

$$\sum_{i=0}^{n-2} \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} + \mathbf{b}_{n-1} \binom{n}{n-1} t^{n-1}(1-t) + \\ + \mathbf{b}_n \binom{n}{n} t^n = \mathbf{b}_{n-1},$$

and it follows that

$$\mathbf{b}_{n-1} = \frac{1}{1 - nt^{n-1}(1-t)} \left(\sum_{i=0}^{n} \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} + \mathbf{b}_n t^n \right).$$

Further we consider that a Bézier curve has a control point $\mathbf{b}_1 \in \mathbf{b}^n(t)$ for all $t \in [0,1]$, then

$$\sum_{i=0}^{n} \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} = \mathbf{b}_1,$$

$$b_0 \binom{n}{0} t^0 (1-t)^n + b_1 \binom{n}{1} t (1-t)^{n-1} + \sum_{i=2}^{n} \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} = \mathbf{b}_1,$$

and finally we obtain

$$\mathbf{b}_1 = \frac{1}{1 - nt^{n-1}(1-t)} \left(\sum_{i=2}^{n} \mathbf{b}_i \binom{n}{i} t^i (1-t)^{n-i} + \mathbf{b}_0 (1-t)^n \right).$$

The following figures show the first class of a cubic constrained Bézier curve.

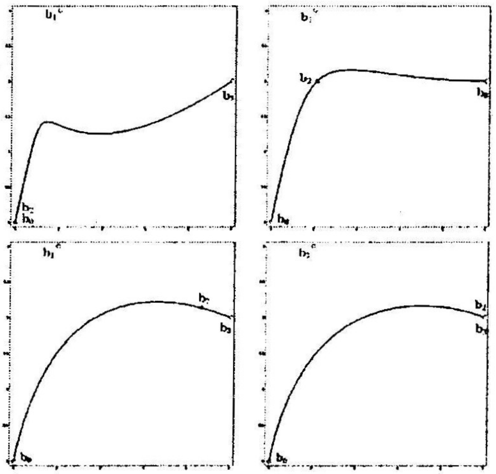

Fig. Constrained Bézier curves at $t = 0, 0.4, 0.8, 1$. The last figure shows the degenerate Bézier curve

REFERENCES

1. Farin G. *Curves and surfaces for CAGD. A practical guide.* – Academic Press, 2002. – 499 p.